Infrastructure Security Games

M. Baykal-Gürsoy, Z. Duan, H. V. Poor and A. Garnaev

Abstract

Infrastructure security against possible attacks involves making decisions under uncertainty. This paper presents game theoretic models of the interaction between an adversary and a first responder in order to study problem of the security within a transportation infrastructure. The risk measure used is based on the consequence of an attack in terms of the number of people affected or the occupancy level of a critical infrastructure, e.g. stations, trains, subway cars, escalators, bridges, etc. The objective of the adversary is to inflict the maximum damage to a transportation network by selecting a set of nodes to attack, while the first responder (emergency management center) allocates resources (emergency personnel or personnel-hours) to the sites of interest in an attempt to find the hidden adversary. This paper considers both static and dynamic, in which the first responder is mobile, games. The unique equilibrium strategy

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pair is given in closed form for the simple static game. For the dynamic game, the equilibrium for the first responder becomes the best patrol policy within the infrastructure. This model uses partially observable Markov decision processes (POMDPs) in which the payoff functions depend on an exogenous people flow, and thus, are time varying. A numerical example illustrating the algorithm is presented to evaluate an equilibrium strategy pair.

I. INTRODUCTION

The September 11, 2001 attacks introduced the term *homeland security* into the public consciousness around the world. In the United States, this term is defined as "a concerted national effort to prevent terrorist attacks within the United States, reduce America's vulnerability to terrorism, and minimize the damage and recover from attacks that do occur" (Homeland Security Act 2002 [1]). Within this effort, protecting critical infrastructure has become an utmost priority for governments [2]. Executive Order 13010 [3] signed by President Clinton in 1996 identifies transportation infrastructure as a critical system supporting the national security and economic well-being of this nation. Moreover, as the Bali and Madrid bombings illustrate, terrorists also target large crowds. Public transit systems, used daily by 32 million mass transit riders in the United States, and places of mass gathering such as shopping malls and stadiums are considered part of the critical infrastructure [4], [5], [6]. Public transit systems by design are open structural environments equipped to move large number of mass transit patrons in an effective and efficient manner. Therefore, mass transit systems are considered *soft targets* similar to the other public places that are inherently vulnerable and susceptible to terrorist attacks and which, because of the continuous hours of service, cannot be closed and secured as may other sectors of the area transportation system [7]. Successful and attempted terrorist attacks throughout the world such as New York, Bali, Madrid, London, Mumbai, Russia, and Norway clearly demonstrate that terrorists' primary mission remains to be mass human casualties in addition to panic and chaos [4]. The threat to any given infrastructural component or "infrastructure" could be substantially reduced by analyzing the risk associated with each transit infrastructure, mitigation planning, and employing best prevention and response policies.

There has been a recent interest in issues related to infrastructure security. A major tool for risk assessment, probabilistic risk analysis (PRA) [8], has also been applied to terrorism risks [9], [10], [11], [12], [13]. On the other hand, the National Research Council [14] has emphasized game theoretic models [15], [16], [17], [18] to counter the need for adaptation to the dynamic behavior of the terrorism events and adversarial decision-making processes of terrorists. One such model, ARMOR [19], [20], [21], [22], [23], casts the *interdiction* problem as a Bayesian Stackelberg game [24], and has been deployed to secure the Los Angeles International Airport. However, this model is static in the sense that it is solved every day with new parameters and the payoff functions for players remain the same throughout the day and the players are assumed to be rational. Aside from the ARMOR game, Brown *et al.* [25] consider various Stackelberg games, while others study network interdiction games [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], secrecy and deception [36], [37], [38], [39], passenger classification [40], [41], and optimal placement of suicide bomber detectors in a grid structure [42]. Hochbaum and Fishbain [43] investigate the allocation of mobile sensors in an urban environment in order to detect *dirty bombs*. Note that the models in [41], [40] and [42] involve only a single controller and not multiple decision makers as in game models.

In this paper, we approach the infrastructure security problem via game theory by modeling it via *hide-and-seek games* [44], [45], [46], [47], [48], [49], [50], [51], [52], [53]. There are two settings for such games: static and dynamic. In the static model, a first responder (emergency-management center) allocates resources (emergency personnel, or personnel-hours) to sites of interest in an attempt to find an object (person or bomb, "adversary") that has been hidden, while the adversary selects a set of best sites to attack. Once the object is hidden, it cannot move during the search process. Similarly, the first responder can act only once. Various different games have been defined for dynamic situations depending on the mobility of the agents. *Search games* [54] involve a mobile defender and an immobile adversary, while *ambush games* [55] have a mobile adversary and an immobile defender, who waits for the adversary to appear. Finally, if both agents are mobile, the games are called pursuit-evasion or *infiltration games* [56]. Most research has focused on the case in which the cells are identical. However, Neuts [57] and later on Sakaguchi [58] consider a zero-sum dynamic search game with node dependent inspection costs. Moreover, there may be a possibility of type 1 error associated to each node, i.e., the probability that the first responder finds the adversary given that the adversary is in the searched node is less than or equal to 1. In general, in the hide-and-seek games there are no attack targets, in fact, the adversary is the target. One exception arises in the *interdiction games* [35], [32] in which the adversary tries to reach a target while the defender tries to prevent the adversary from reaching the target, thereby protecting the target. Recently, interdiction games with various targets have been considered. Such games are called *protection games* (please see [59] and the references there in).

In this paper, we study protection games. Focusing on severe attacks, we consider the loss of human life as the consequence of the attack, i.e., the payoff to the adversary. This measure typically depends on the occupancy level of the facility and we assume that the occupancy level can be estimated over time. Hence the crowds are the targets in this game and since they are moving over time they are dynamically moving targets. The static version of this game becomes a simple zero-sum game related to the one considered by Neuts [57] and Sakaguchi [58]. However, contrary to their case we observe that in our game a continuum equilibrium for the adversary may exist under certain conditions. In the dynamic game model, we assume that the first responder can move among the nodes to search for a hidden immobile adversary. This game is called as a *patrolling* game [46], and [59] with the additional feature of multiple mobile targets. We sometimes refer to resources allocated to the nodes also as first responders. The main idea here is that if the emergency-management center has a finite number of first responders, it then allocates fractions of first responders to the nodes. Throughout we use first responder and defender, and, respectively, adversary and attacker, synonymously.

Contributions of this paper are itemized below.

- A new static game is introduced that considers the occupancy of a node as the payoff to the adversary. This game is shown to have a unique equilibrium for the first responder in closed form. However, the adversary may have continuum of equilibria, also given in closed form. The equilibria are of threshold type, i.e., the resources are allocated to the nodes with occupancy higher than a threshold value.
- A novel protection game with dynamically moving targets is introduced, and its solution algorithm through an illustrative example is provided.

The structure of the paper is as follows. In the next section, we briefly review the relevant literature. In Section II, we consider the static game and present the unique equilibrium in closed form. In Section III, a people flow model is introduced. In Section IV, a dynamic game between an immobile adversary and a mobile first responder is discussed. In Section V, we present a numerical example for the dynamic game. Finally, further applications and future research directions are discussed in Section VI.

II. STATIC INFRASTRUCTURE GAME MODEL

In this section, we consider the one-step security problem. The adversary and the first responder simultaneously choose their strategies over the potential sites. Payoff matrices for both responder and adversary are based on the occupancy level of each site in the infrastructure. Even when both rivals are at the same site, there is a probability that the first responder may not detect the adversary. We assume that the infrastructure can be partitioned into nodes. This could be achieved, for example, as described in [60], [61]. We further assume that the impact of an attack will be based upon the occupancy level of the specific node at which the attack happens and can only endanger the people at that node. People in neighboring nodes will not be hurt directly due to this attack. We assume that the probability of detection, and the occupancy of each node, are known to both rivals.

Although we represent the infrastructure in the figures as an m by n grid, it will be considered as an undirected graph with $N = mn$ nodes, which the responder and the adversary can occupy (possible actions for both players). Bold case letters represent vectors; for example, the occupancy vector is denoted as $\boldsymbol{O}$ where O_i is the occupancy level of node $i = 1, 2, \ldots, N$, that also gives the expected number of casualties at node i.

Let (i, j) denote the location of the first responder and the adversary, respectively. In the case in which both of them are at the same node, (i, i), the adversary can be found by the responder with detection probability given by $P_D(i, i) \stackrel{\triangle}{=} d_i$, otherwise $P_D(i, j) \stackrel{\triangle}{=} 0$ for $i \neq j$. Therefore, the (i, j)-th component of the payoff matrix for the responder when the responder is searching node i and the adversary is attacking node j is, $r_{ij} = -(1 - P_D(i, j)O_j)$ for all $i, j = 1, \ldots, N$. Under the above assumptions, the payoff matrix R for the first responder is an $N \times N$ matrix with the following elements for the action pair (i, j) of the first responder and the adversary, respectively:

$$R = \begin{array}{c} i \setminus j \\ 1 \\ 2 \\ \vdots \\ N \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & N \end{array} \\ \begin{pmatrix} -(1-d_1)O_1 & -O_2 & \cdots & -O_N \\ -O_1 & -(1-d_2)O_2 & \cdots & -O_N \\ \vdots & \vdots & \ddots & \vdots \\ -O_1 & -O_2 & \cdots & -(1-d_N)O_N \end{pmatrix} \end{array},$$

with $d_i \in (0, 1)$.

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_N)^T$ be a (mixed) strategy column vector for the first responder where x_i is the probability of searching node i. Clearly, $\sum_{i=1}^N x_i = 1$, and $x_i \geq 0$, for all $i \in \{1, \ldots, N\}$. A (mixed) strategy for the adversary, $\boldsymbol{y}$ is similarly defined, where y_i is the probability of attacking node i. The expected payoff to the first responder if the rivals apply mixed strategies $\boldsymbol{x}$ and $\boldsymbol{y}$ is given as follows:

$$v(\boldsymbol{x}, \boldsymbol{y}) = -\sum_{i=1}^N x_i \left[\sum_{j=1}^N O_j y_j - d_i O_i y_i \right]. \tag{1}$$

The payoff to the adversary is $-v(\boldsymbol{x}, \boldsymbol{y})$, thus giving a zero-sum game. Note that $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ is a saddle point (Nash equilibrium) if and only if the following inequalities hold:

$$v(\boldsymbol{x}, \boldsymbol{y}^*) \leq v(\boldsymbol{x}^*, \boldsymbol{y}^*) \leq v(\boldsymbol{x}^*, \boldsymbol{y}) \text{ for any } (\boldsymbol{x}, \boldsymbol{y}). \tag{2}$$

Also, $v = v(\boldsymbol{x}^*, \boldsymbol{y}^*)$ is the value of the game.

This game is closely related to a multistage game of Neuts [57] and a two-sided search game suggested by Sakaguchi [58], but our game has one interesting particular phenomena as is shown in the next theorem; namely, under particular conditions continuum equilibria could arise.

For the sake of brevity, we assume that

$$O_1 > O_2 > \cdots > O_N. \tag{3}$$

Next we present a brief version of the theorem that shows the game has a unique equilibrium in closed form under certain conditions. However, the adversary may have continuum of equilibria, also given in closed form under some conditions. The equilibria are of threshold type, i.e., the resources are allocated to the nodes with value higher than a threshold as given below. The detailed statement and proof of the theorem can be found in the appendix.

Theorem 1*: If $v^* \neq -O_k$, then the game has the unique equilibrium $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ given in terms of the index $k \in \{1, \ldots, N\}$ such that $\varphi_k \leq 1 < \varphi_{k+1}$, where $\{\varphi_i\}$ is a strictly increasing sequence defined as $\varphi_i = \sum_{j=1}^{i} (O_j - O_i)/d_j O_j$, for $i \in \{1, \ldots, N,\}$ and $\varphi_{N+1} = \infty$.*

The strategy of the defender is of threshold type given by $x_i^* = \dfrac{1/(d_i O_i)}{\sum_{j=1}^{k} 1/(d_j O_j)} \left(1 - \sum_{j=1}^{k} \dfrac{O_j - O_i}{d_j O_j}\right)$ *for* $i \leq k$, *and* 0 *if* $i \geq k$.

The strategy of the adversary is also of threshold type given by $y_i^* = 1/(d_i O_i) \sum_{j=1}^{k} 1/(d_j O_j)$, *for* $i \leq k$, *and* 0 *otherwise. The value of the game is given by* $v^* = \dfrac{1 - \sum_{j=1}^{k} 1/d_j}{\sum_{j=1}^{k} 1/(d_j O_j)}$.

Remark 1 *If all the nodes have the same detection probability, i.e.* $d_i = d$ *for any* i, *then* x_i *is decreasing, i.e.* $x_1 > x_2 > \ldots > x_k$, *and meanwhile* y_i *is increasing, i.e.* $y_1 < y_2 < \ldots < y_k$.

Remark 2

Next, we discuss the exogenous people flow model that will be used in the dynamic game in order to estimate the occupancy level of each node.

III. People Flow Model

We first develop an exogenous people flow model that influences the decision making of the game players. A number of researchers have used simulation models to describe the characteristics of pedestrian flow [62], [63], [64]. Yue *et al.* [62] introduce a simulation model based on cellular automata on the square lattice with two-way and four-way pedestrian flow. In this simulation framework, pedestrian movement is more flexible and adaptive to dynamic conditions than in vehicular flow. Hanish *et al.* [63] develop an online simulation tool for pedestrian flow in large public buildings, such as train stations, airports, shopping centers, etc. There is also research that concentrates on occupancy estimation [65], [66], [67]. Deng *et al.* [65] introduce a sensor-utility-network (SUN) method for occupancy estimation in buildings. Other studies have focused on the pedestrian flow in public buildings following special events, such as football matches, emergency fires, and terrorist attacks, etc., with crowd control and optimal evacuation as the main objectives [68], [69], [70], [71], [72], [73]. Alternating periods of congestion and slow movement dominate these cases, and most research on this topic relies on simulation models as in [74] and [71].

For our purposes, we model people flow in a public building as a linear, stochastic dynamic system, and assume that some sensory information is available to be used in correcting the occupancy estimates. In this paper, we will not take into account the effects of special events, and also we will not consider crowd control problems.

At the microscopic level, people move as if they were in an open queuing network where each node is considered as a queueing station. Time is discretized, and the time horizon is finite and is equal to T. Nodes may have external arrivals and departures, from and to the outside, respectively, if there is direct connection to the outside, such as entrance doors, or train platforms from which people get on or off the train. Other nodes in the building could be ticket offices, waiting rooms, food courts, shops, hallways, etc.

For those nodes with entrances, an arrival rate is estimated, captured in a vector $\boldsymbol{\lambda} \in \mathcal{R}^N$ whose elements are arrival rates for each node per unit time. Arrival rates may be time varying, as $\boldsymbol{\lambda}(m)$, representing peak and off peak hours during the day. At each time period, people move from one node to the other according to the probabilities given by the routing matrix, $\boldsymbol{F}$. We assume that pedestrians are all similar, and thus, they all have the same routing probability. These assumptions result in the following stochastic linear dynamic system of equations representing the pedestrian flow:

$$\boldsymbol{O_{m+1}} = \boldsymbol{F}^T \cdot \boldsymbol{O}_m + \boldsymbol{\lambda}_{m+1} + \boldsymbol{W_{m+1}}, \qquad \boldsymbol{W_{m+1}} \sim \mathcal{N}(0, Q)$$

$$\boldsymbol{Z_m} = H \cdot \boldsymbol{O_m} + \boldsymbol{\Gamma_m}, \qquad \boldsymbol{\Gamma_m} \sim \mathcal{N}(0, \Xi)$$

The first equation is the state equation, in which $\boldsymbol{O_m}$ denotes the occupancy vector, and $\boldsymbol{W_m}$ denotes the process noise at time m, which is assumed to be normally distributed with covariance matrix Q. The second equation is the observation equation. $\boldsymbol{Z_m} \in \mathcal{R}^M$ is the measurement vector of actual occupancies at time m, H is the measurement matrix, and $\boldsymbol{\Gamma}$ denotes the measurement noise which is normally distributed with covariance matrix Ξ. These measurements are obtained from video cameras, sensors, and other inspection methods. Here M may be less than N, meaning that not all node occupancies may be available. However, we assume that the system is observable.

A Kalman filter is used to predict and correct the occupancy level estimates. The *Kalman filter* is an efficient recursive filter that estimates the state of a linear dynamic system from a series of noisy measurements [75], [76], [77], [78]. At each time period, the responder will observe $\boldsymbol{Z}_m$, the occupancy vector, and then use these measurements to correct occupancy level forecasts. The equations for the Kalman filter are as follows:

$$\hat{\boldsymbol{O}}^-_{m+1} = \boldsymbol{F}^T \cdot \hat{\boldsymbol{O}}_m + \boldsymbol{\lambda_{m+1}}; \tag{4}$$

$$\hat{\boldsymbol{O}}_{m+1} = \hat{\boldsymbol{O}}^-_{m+1} + K_{m+1} \cdot (\boldsymbol{Z_{m+1}} - H\hat{\boldsymbol{O}}^-_{m+1}); \tag{5}$$

$$P_{m+1}^{-} = \boldsymbol{F}^T \cdot P_m \cdot \boldsymbol{F} + Q; \tag{6}$$

$$P_{m+1} = (I - K_{m+1}H) \cdot P_{m+1}^{-} \tag{7}$$

where P_m and P_m^{-} are the posterior and prior estimation error covariance matrices, respectively. K_{m+1} is the Kalman gain given by

$$K_{m+1} = P_{m+1}^{-} H^T \cdot (H P_{m+1}^{-} H^T + \Xi)^{-1}.$$

In these equations, $\hat{\boldsymbol{O}}_{m+1}^{-}$ and P_{m+1}^{-} are the forecast values that are used to determine the initial position and initial patrol sequence. $\hat{\boldsymbol{O}}_{m+1}$ and P_{m+1} are the corrected values after each measurement. They are used at the beginning of each time period to reevaluate and update the original patrol sequence.

Example 1 *Fig. 1 shows an example representing the node occupancies in a* 2×2 *grid. This figure is*

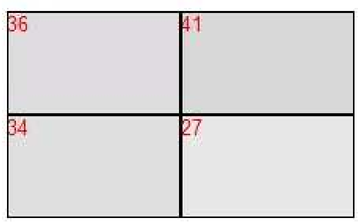

Fig. 1. Node Occupancy

taken from the simulation package we have developed for this application. The number in each node represents the current occupancy level. Nodes are numbered from 1 to 4, the top left node being 1, and the bottom right node being 4. Assuming that there are entrances at nodes 1 and 4, we have the following arrival rate vector:

$$\boldsymbol{\lambda}^T = (\lambda_1, \lambda_2, \lambda_3, \lambda_4) = (\lambda_1, 0, 0, \lambda_4),$$

where bold letters denote vectors, and the superscript T *denotes the vector transpose. Below is a* 4×4

matrix representing the routing matrix.

$$\boldsymbol{F} = \begin{bmatrix} 0.2 & 0.3 & 0.3 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \\ 0.4 & 0 & 0.1 & 0.5 \\ 0 & 0.3 & 0.2 & 0.2 \end{bmatrix}$$

From this matrix, notice that people leave the infrastructure from node 1 with probability $1 - 0.2 - 0.3 - 0.3 = 0.2$*, will go to node 2 and 3 with the same probability 0.3, and will stay at node 1 with 0.2 probability. Similarly, people leave node 4 with probability* $1 - 0.3 - 0.2 - 0.2 = 0.3$*. Those nodes in which people can enter or leave are the entrance doors, as well as the train platforms where they can get on and off the trains.*

IV. DYNAMIC SECURITY GAME

In this section, we consider a mobile first responder dynamically choosing nodes to search for an immobile adversary. The first responder's objective is to develop a "best" patrol strategy to find the adversary with maximum reward or minimum cost. We assume that if the adversary is not caught within a finite time, say T, the adversary will launch the attack and destroy occupants in the node s/he is in at time T. The first responder does not know the exact location of the adversary. Furthermore, the responder may not also have the current occupancy information. However, some sensory data are assumed to be available in order to develop for example, the above discussed people flow model, and make accurate estimates. Next, we describe the methodology to obtain best strategies for both players.

A. Strategy Development

Occupancy estimates together with the detection probabilities establish the performance measure on the infrastructure grid. Note that only the first responder is mobile. Thus, after choosing the initial locations the first responder can patrol the premises to find the adversary while the adversary remains at its initial location. Next, we describe the first responder's patrol strategy starting from an initial position. Then, we will discuss the first responder's and adversary's strategies for picking the best initial location.

1) First Responder's Strategy: Consider discrete time periods $\{m = 0, 1, 2, \ldots, T\}$ and the grid structure representing the infrastructure. The state of the system is given by $\{l^f_m, l^a_m, \boldsymbol{O_m}, P_m\}$, where l^f_m and l^a_m denote the position of the first responder and the adversary, respectively, at time m. Since the adversary is immobile, $l^a_m = l^a$. However, the adversary's location cannot be observed and the first responder has information only about his/her own location, i.e., l^f_m and P_m; thus, arises the need to use the POMDP (partially observable Markov decision process) model to solve this problem.

Let the first responder's belief state be $\boldsymbol{b_m} = \{l^f_m, \boldsymbol{p^a_m}, \boldsymbol{\hat{O}_m}, P_m\}$. Here, $\boldsymbol{p^a_m}$ is the vector of belief probabilities of the adversary's location. $\boldsymbol{\hat{O}_m}$ is the estimated occupancy vector at time m. The value function for the responder at time $m = 0, \ldots, T-1$ is given by

$$V^f_m(\boldsymbol{b_m}) = \max_{k \in A^f\{l^f_m\}} \left\{ r(\boldsymbol{b_m}, k) + \gamma \cdot (1 - Pr\{\mathrm{D}|\boldsymbol{b_m}, k\}) \cdot V^f_{m+1}(l^d_{m+1}, \boldsymbol{p^a_{m+1}}, \boldsymbol{\hat{O}^-_{m+1}}, P^-_{m+1}) \right\}, \tag{8}$$

where $r(\boldsymbol{b_m}, k)$ denotes the one-step expected reward function for the responder in belief state $\boldsymbol{b_m}$ when action k is applied, and γ denotes the discount factor with $(\gamma \leq 1)$, k denotes the responder's action, i.e., the next node in the patrol route, and $A^f\{l^f_m\}$ denotes the set of responder's possible actions at the next time period when the responder's current location is l^f_m. $Pr\{\mathrm{D}|\boldsymbol{b_m}, k\}$ is the probability of detecting the adversary given the belief state $\boldsymbol{b_m}$ and applying action k. We assume that when the responder successfully finds the adversary, the game will end, and no more rewards will be earned. $\boldsymbol{\hat{O}^-_{m+1}}$ and P^-_{m+1} are the corresponding forecast values of the occupancy levels and the error covariance matrix. The one-step expected reward function is written as

$$r(\boldsymbol{b_m}, k) = Pr\{\mathrm{D}|\boldsymbol{b_m}, k\}(\hat{O}_{m,k} + C), \tag{9}$$

where C is the terminal reward for apprehending the adversary. Here $Pr\{\mathrm{D}|\boldsymbol{b_m}, k\}$ is equal to

$$Pr\{\mathrm{D}|\boldsymbol{b_m}, k\} = d_k p_m^a(k). \tag{10}$$

Since the attack will materialize at the end of period T, the terminal value function is given as the negative of the expected casualty due to this attack,

$$V_T^f(\boldsymbol{b_T}) = -\sum_{k=1}^{N} p_T^a(k) \cdot \hat{O}_{T,k}. \tag{11}$$

Note that transitions from $\boldsymbol{\hat{O}_m}$ and P_m to $\boldsymbol{\hat{O}_{m+1}}$ and P_{m+1}, respectively, are associated with the people flow model and are given through equations (4) to (7) deterministically. Also because action k identifies the location of the first responder in the next time period, the transition probability for POMDP from state $\boldsymbol{b_m}$ to $\boldsymbol{b_{m+1}} = \{k, p_{m+1}^a, \boldsymbol{\hat{O}_{m+1}}, P_{m+1}\}$ is as follows:

$$Pr\left\{\boldsymbol{b_{m+1}}|\boldsymbol{b_m}, k\right\} = \begin{cases} 1 - Pr\{\mathrm{D}|\boldsymbol{b_m}, k\}, & \text{for } \boldsymbol{p_{m+1}^a} \text{ as in equations (13) and (14)} \\ Pr\{\mathrm{D}|\boldsymbol{b_m}, k\}, & \text{for } \boldsymbol{p_{m+1}^a} = \boldsymbol{e_k}. \end{cases} \tag{12}$$

In both cases above, the first responder moves to node k to search for the adversary, so the location of the first responder at time $m + 1$ is node k. In the first case, the first responder is not able to detect the adversary in node k, so the belief probabilities will be updated through equations (13) and (14) given below. In the second case, the responder finds the adversary in node k, so the belief probability vector is $\boldsymbol{e_k}$, the kth coordinate vector. The dynamic security game terminates after finding the adversary, at which point all future value functions V_m^f will be 0, and so the corresponding term in equation (8) is eliminated.

We can write the Bayesian update equations for belief probabilities. If the responder fails to detect the adversary at time m in node k, then the belief probability of node k is reduced while the belief probabilities of other nodes are increased as given by

$$p_{m+1}^a(k) = Pr\left\{l_{m+1}^a = k|\boldsymbol{p_m^a}, \text{Detection Failed}\right\} = \frac{(1 - d_k) \cdot p_m^a(k)}{1 - d_k \cdot p_m^a(k)}, \tag{13}$$

$$p^a_{m+1}(j) = \frac{p^a_m(j)}{1 - d_k \cdot p^a_m(k)} \qquad \text{for } j \neq k,, \tag{14}$$

where, d_k is the detection probability for node k.

The Bellman equation (8), representing the POMDP, together with the terminal value function in equation (11), can be solved using dynamic programming algorithms. The solution provides a sequence of actions (patrol strategy) for the first responder, as well as the total expected risk or reward for the responder, i.e., the value function $V^f_m(\boldsymbol{b_m})$. In most cases, given the initial position of the first responder, the patrol strategy is deterministic. However, due to different detection results and changing occupancy levels, this patrol strategy will be reevaluated after each search based on the current flow information.

Next, we will discuss the initial position game between the responder and the adversary.

2) Responder and Adversary's Initial Position Game: The responder and the adversary must decide on their initial positions first. This is a static game between two players. To obtain the elements of the reward matrix, say the (i, j)th element, first, the POMDP model generates an optimal patrol sequence starting from the initial position of the first responder (node i). Then, given the initial position of the adversary (node j), the expected rewards are calculated based on this patrol sequence. The expected rewards will be different from the value function obtained in the POMDP. The value function in the POMDP is the expected reward for every possible location of the adversary, however, the rewards we obtain for this static game are the expected rewards for the fixed location combination (i, j) of the first responder and the adversary. In this initial position game, since future sensory information is not available at the time, forecast occupancy levels are used. The elements of the reward matrix are obtained as follows:

$$R_f(i,j) = \sum_{m=0}^{T-1} Pr\{\text{D at time } m|\boldsymbol{S}, l^a = j\}\gamma^m \hat{O}_{m,j} - Pr\{\text{No D within } T|\boldsymbol{S}, l^a = j\}\gamma^T \hat{O}_{T,j}, \tag{15}$$

where $\boldsymbol{S}$ is the patrol sequence obtained through the POMDP with $S(n)$ denoting the node searched at time n, and $S(0) = i$. $Pr\{\text{No D within } T|\boldsymbol{S}, l^a = j\}$ denotes the probability that the adversary will not be detected, thus, the attack at time T will materialize, and is given as

$$Pr\{\text{No D within } T|\boldsymbol{S}, l^a = j\} = \prod_{n=0}^{m-1} \left[1 - I\{S(n) = j\}d_j\right],$$

where $I\{\cdot\}$ denotes the indicator function for the event represented inside the parentheses, i.e., if the event happens then the function takes value 1, otherwise its value is zero. $Pr\{\text{D at time } m|\boldsymbol{S}, l^a = j\}$ denotes the probability that detection of the adversary happens exactly at time m, given the first responder's patrol sequence and adversary's position, which can be written as

$$\begin{aligned} Pr\{\text{D at time } m|\boldsymbol{S}, l^a = j\} &= Pr\{\text{No D within } m-1|\boldsymbol{S}, l^a = j\} I\{S(m) = j\}d_j \\ &= \prod_{n=0}^{m-1} \left[1 - I\{S(n) = j\}d_j\right] \cdot I\{S(m) = j\}d_j, \qquad (0 < m < T), \\ Pr\{\text{D at time } 0|\boldsymbol{S}, l^a = j\} &= I\{S(0) = j\}d_j. \end{aligned}$$

This static game is then solved to obtain the mixed strategy of initial position for the first responder and the adversary. Note that with this randomized initial position for the first responder, the optimal patrol sequence will also be randomized.

V. Illustrative Example

In this section, we use an example to explain details of the POMDP model and initial position game. First, we describe the POMDP procedure, and then we present the results of the initial position game between the responder and the adversary. Together, an optimal patrol strategy for the first responder is developed.

A 3×3 grid infrastructure is considered, and nodes are numbered from 1 to 9 from top left to bottom right. The total number of time periods is $T = 7$. The flow transition probabilities are given below as a 9×9 matrix:

$$\boldsymbol{F} = \begin{bmatrix} 0.1 & 0.3 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0.1 & 0.2 & 0.4 & 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.1 & 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.1 & 0.4 & 0 & 0.3 & 0 & 0 \\ 0 & 0.1 & 0 & 0.2 & 0.4 & 0.1 & 0 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0 & 0.3 & 0.1 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0.2 & 0 & 0 & 0.1 & 0.3 & 0 \\ 0 & 0 & 0 & 0 & 0.3 & 0 & 0.1 & 0.2 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0.2 & 0.1 \end{bmatrix}.$$

Arrival rates to each node from outside the infrastructure is as follows:

$$\boldsymbol{\lambda} = [25, \quad 0, \quad 20, \quad 0, \quad 0, \quad 0, \quad 10, \quad 0, \quad 15]^T.$$

The numbers of arrivals per period in the various nodes are Poisson random variables with the above arrival rates. From this, we can see that there are gates at nodes 1, 3, 7 and 9 in this facility. Initial occupancy levels are given as

$$\boldsymbol{O_0} = [50, \quad 32, \quad 41, \quad 42, \quad 80, \quad 35, \quad 51, \quad 45, \quad 39]^T.$$

Q is a diagonal matrix with diagonal elements given as

$$[25, \quad 36, \quad 50, \quad 40, \quad 37, \quad 28, \quad 48, \quad 35, \quad 40].$$

Ξ is a diagonal matrix with diagonal elements given as

$$[10, \quad 6, \quad 8, \quad 7, \quad 8, \quad 9, \quad 4, \quad 15, \quad 10].$$

P_0 is a diagonal matrix with diagonal elements given as

$$[4, \quad 5, \quad 5, \quad 3, \quad 9, \quad 2, \quad 5, \quad 4, \quad 3].$$

And H is an identity matrix in this example, which means that the measurements are observations of the actual occupancy levels.

The detection probabilities for each node are

$$\boldsymbol{P}_D = [0.8, \quad 0.9, \quad 0.7, \quad 0.9, \quad 0.6, \quad 0.7, \quad 0.8, \quad 0.75, \quad 0.8]^T.$$

The prior adversary-location belief probabilities are equally distributed among 9 nodes.

Detection costs are all assumed to be negligible, and the detection terminal reward is also zero. The discount factor is $\gamma = 1$, since this is a short period game, it is appropriate to assume that penalty and reward are not discounted in such a short period.

A. POMDP Results

First, assuming that the initial position of the responder is known, we use the POMDP model to optimize the responder's patrol strategy for each time period. At each time period, the responder will observe $\boldsymbol{Z}_m$, the occupancy vector through video camera, sensors or other methods, and then use these measurements to correct occupancy level forecasts. The actual occupancy levels and measurements for each time period are given as follows:

$$O = \begin{bmatrix} 50 & 97 & 52 & 42 & 47 & 45 & 46 & 39 \\ 32 & 39 & 67 & 63 & 62 & 65 & 63 & 51 \\ 41 & 92 & 59 & 79 & 54 & 64 & 50 & 58 \\ 42 & 54 & 44 & 44 & 48 & 43 & 42 & 53 \\ 80 & 79 & 91 & 111 & 113 & 115 & 113 & 112 \\ 35 & 35 & 56 & 45 & 57 & 44 & 50 & 43 \\ 51 & 60 & 34 & 32 & 16 & 45 & 27 & 30 \\ 45 & 42 & 59 & 50 & 49 & 53 & 53 & 53 \\ 39 & 71 & 50 & 74 & 64 & 64 & 59 & 59 \end{bmatrix}, \qquad Z = \begin{bmatrix} 103 & 53 & 39 & 49 & 43 & 44 & 42 \\ 37 & 70 & 63 & 65 & 65 & 65 & 51 \\ 90 & 57 & 79 & 54 & 63 & 49 & 60 \\ 57 & 47 & 44 & 48 & 40 & 42 & 55 \\ 77 & 92 & 107 & 109 & 113 & 111 & 110 \\ 37 & 59 & 47 & 60 & 46 & 52 & 40 \\ 61 & 37 & 32 & 15 & 44 & 28 & 32 \\ 42 & 56 & 42 & 45 & 51 & 57 & 57 \\ 73 & 54 & 75 & 67 & 73 & 61 & 61 \end{bmatrix} \tag{16}$$

The actual occupancy levels O is a 9×8 matrix, formed by column vectors $O_0, O_1, \ldots, O_7$. Measurements $\boldsymbol{Z_m}$ for each time period are given in the matrix Z (9×7), and column m of Z, $\boldsymbol{Z_m}$, represents the measurements on actual occupancy levels at time m.

Let the first responder start from node 4. If measurements in Z are not available, then the optimal patrol sequence for the responder is $\{4, 5, 8, 9, 6, 3, 2\}$. However, if they are available, then the first responder will correct the forecasts on occupancy levels, and then use the POMDP to develop a new patrol sequence which will only be used for the next move. The optimal patrol sequence in this case is $\{4, 5, 2, 3, 6, 9, 8\}$. This is different from the previous patrol sequence due to the Kalman filter correction procedure. Both patrol sequences will check the same nodes, but in different orders, and thus will generate different expected rewards for the first responder.

The following two matrices contain patrol sequences for the first responder, starting from every initial node, either without or with actual measurements. The first matrix gives the patrol sequences when Z is not available, and the second matrix are patrol sequences when Z is available. Each row represents one patrol sequence.

$$\begin{bmatrix} 1 & 4 & 5 & 2 & 3 & 6 & 9 \\ 2 & 1 & 4 & 5 & 8 & 9 & 6 \\ 3 & 2 & 1 & 4 & 5 & 8 & 9 \\ 4 & 5 & 8 & 9 & 6 & 3 & 2 \\ 5 & 8 & 9 & 6 & 3 & 2 & 1 \\ 6 & 9 & 8 & 5 & 4 & 1 & 2 \\ 7 & 4 & 1 & 2 & 5 & 8 & 9 \\ 8 & 9 & 6 & 3 & 2 & 5 & 4 \\ 9 & 8 & 5 & 4 & 1 & 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 4 & 5 & 2 & 3 & 6 & 9 \\ 2 & 1 & 4 & 5 & 6 & 9 & 8 \\ 3 & 2 & 1 & 4 & 5 & 8 & 9 \\ 4 & 5 & 2 & 3 & 6 & 9 & 8 \\ 5 & 8 & 9 & 6 & 3 & 2 & 1 \\ 6 & 9 & 8 & 5 & 2 & 1 & 4 \\ 7 & 4 & 1 & 2 & 5 & 8 & 9 \\ 8 & 9 & 6 & 3 & 2 & 5 & 4 \\ 9 & 8 & 5 & 4 & 1 & 2 & 3 \end{bmatrix} \tag{17}$$

Most of the patrol sequences are similar under both cases, except starting from initial nodes 2, 4, and 6. The patrol sequences will be updated due to the information obtained through measurements. Updated patrol sequences usually generate better rewards for the first responder.

B. Initial Position Game

When the responder and the adversary decide on their initial positions, the responder does not have the information about future measurements provided in Z. So Z is not needed in this procedure. For each initial position of the first responder, the optimal patrol sequence is already given in the first matrix of equation (17), based only on the forecast occupancy levels. The people flow model forecasts the future occupancy levels to provide expected risk measures throughout the finite time period T.

Given the position of both the first responder and the adversary, and the patrol sequence, the expected rewards can be easily calculated by using equation (15). So, R_f, the reward matrix for the first responder in the initial position game can be built.

Given the simplest case, in which the initial position game is a zero-sum game, the following Nash Equilibrium strategies for the first responder and the adversary are obtained:

$$X^* = (0.1464, \quad 0.0000, \quad 0.0000, \quad 0.4776, \quad 0.0000, \quad 0.0000, \quad 0.3556, \quad 0.0000, \quad 0.0204)^T$$

$$Y^* = (0.0014, \quad 0.0000, \quad 0.4691, \quad 0.0000, \quad 0.0000, \quad 0.0038, \quad 0.5256, \quad 0.0000, \quad 0.0000)^T.$$

One can see that the first responder can choose from nodes $1, 4, 7,$ and 9 as its initial position, and the adversary can choose from nodes $1, 3, 6,$ and 7 as its attacking position. The game value for this initial position game is -5.1602. This means that the expected reward for the first responder is negative, so the first responder needs to improve the probability of detection or deploy more personnel to perform the search.

Note that this game value is calculated based on the forecast occupancy levels without any correction. To calculate the actual expected reward of the first responder under strategy (X^*, Y^*), the actual occupancy levels O in equation (16) are used. Both the updated and non-updated patrol sequences are given in equation (16). The actual expected reward obtained under updated patrol sequence is -3.2757, while it is -5.6281 when updates are not available. Clearly, the updated patrol sequence is better. However, this is not always the case; there exist some cases in which updated sequences generate less expected reward than non-updated sequences. Generally speaking, when people flow experiences unusual shocks, such as sudden influx or outflux of people, the updated patrol sequence will work much better.

On the other hand, if the first responder improves the detection probability to 1 for all nodes, the actual expected rewards for non-updated and updated patrol sequences become 3.8816 and 3.9608, respectively. Updated patrol sequences generate slightly better reward for the first responder in this case too. They both are much better than the original case with lower detection probabilities. Of course, in this case, non-updated and updated patrol sequences, and the mixed strategy for the initial position (X^*, Y^*), will all be different from the original case.

VI. Discussion and future research

In this paper, we have considered static and dynamic game models for the infrastructure security problem. In these models, rewards and costs are based upon the occupancy level at each location in the infrastructure. Our models can be used in obtaining real time strategies for infrastructure security personnel.

For the static game, we have proved certain properties of the equilibrium. While for the dynamic game in which the first responder is mobile we have presented a solution methodology that is based on the POMDP model. Throughout, examples have been provided.

Next, we plan to study the two-controller resource allocation problem in which a number of sites (targets) are attacked by the adversary and are defended by the first responders. Depending on the players' objectives, such a problem can be modeled as a zero-sum stochastic game [79], [80], [81], [82], or a Nash game [83]. Our planned approach is to consider discrete and known environment, and incorporate risk measures into the objective function.

APPENDIX A

THEOREM AND PROOF

Theorem 1 *(a) The game has the unique equilibrium* $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ *for* $v^* \neq -O_k$ *where* $k \in \{1, \ldots, N\}$ *is such that*

$$\varphi_k \leq 1 < \varphi_{k+1} \tag{18}$$

where $\{\varphi_i\}$ *is strictly increasing sequence given by*

$$\varphi_i = \sum_{j=1}^{i} \frac{O_j - O_i}{d_j O_j}, \quad i \in \{1, \ldots, N\} \tag{19}$$

and $\varphi_{N+1} = \infty$ *and*

$$x_i^* = \begin{cases} \dfrac{1/(d_i O_i)}{\displaystyle\sum_{j=1}^{k} 1/(d_j O_j)} \left(1 - \displaystyle\sum_{j=1}^{k} \frac{O_j - O_i}{d_j O_j}\right), & i \leq k, \\ \\ 0, & i \geq k+1, \end{cases} \tag{20}$$

and

$$y_i^* = \begin{cases} \dfrac{1/(d_i O_i)}{\displaystyle\sum_{j=1}^{k} 1/(d_j O_j)}, & i \leq k, \\ \\ 0, & i \geq k+1. \end{cases} \tag{21}$$

The value of the game is given by

$$v^* = \frac{1 - \sum_{j=1}^{k} 1/d_j}{\sum_{j=1}^{k} 1/(d_j O_j)}. \tag{22}$$

(b) If $v^* = -O_k$ *then* $-O_k$ *is the value of the game. The first responder has the unique equilibrium strategy*

$$x_i^* = \frac{1}{d_i}\left[1 - \frac{O_k}{O_i}\right]_+, i \in \{1, \ldots N\}; \tag{23}$$

meanwhile the adversary has continuum equilibrium strategies

$$y_i^* = \begin{cases} \frac{\epsilon}{d_i O_i} & i \leq k-1, \\ 1 - \epsilon \sum_{j=1}^{k-1} \frac{1}{d_j O_j}, & i = k, \\ 0, & i \geq k+1, \end{cases} \tag{24}$$

for any

$$\epsilon \in \left[\frac{1}{\sum\limits_{j=1}^{k} (1/(d_j O_j))}, \frac{1}{\sum\limits_{j=1}^{k-1} (1/(d_j O_j))} \right].$$

Proof: We know that the equilibrium strategies for this zero-sum game is given by the optimum solution, $(\boldsymbol{x}^*, \boldsymbol{y}^*)$, to the following primal and dual problem pair:

$$(P) \quad max \quad v \qquad\qquad (D) \quad min \quad w \tag{25}$$

$$s.t. \quad R^T \boldsymbol{x} \geq v, \quad (\boldsymbol{y}) \qquad\qquad s.t. \quad R\boldsymbol{y} \leq w, \quad (\boldsymbol{x}) \tag{26}$$

$$\sum_{i=1}^{N} x_i = 1, \quad (w) \qquad\qquad \sum_{j=1}^{N} y_j = 1, \quad (v) \tag{27}$$

$$\boldsymbol{x} \geq 0. \qquad\qquad \boldsymbol{y} \geq 0. \tag{28}$$

(a) If $y_i^* > 0$, then $\boldsymbol{R}^T(i, \cdot) x_i^* = v^* \neq O_k$, where, $\boldsymbol{R}^T(i, \cdot) = \boldsymbol{R}(\cdot, i)^T$. This implies,

$$-O_i + d_j O_i x_i^* = v^*, \tag{29}$$

giving $x_i^* = \frac{O_i + v^*}{d_i O_i}$. The normalization equation (27) gives the value of the game (22) for some $k \in 1, 2, \ldots, N$. Substituting v^* value in (29) provides the optimal policy for the first responder as given in (20). Note that under assumption (3) the following holds

$$\left(1 - \sum_{j=1}^{k} \frac{O_j - O_i}{d_j O_j}\right) \geq \left(1 - \sum_{j=1}^{k} \frac{O_j - O_k}{d_j O_j}\right), \quad i \leq k, \tag{30}$$

and the right hand side parenthesis is decreasing with respect to k. Thus, the value of k is constructed from the non-negativity of x_i^*'s for $i \leq k$ in equations (20) and (30).

If $x_i^* > 0$, then by complementary slackness, $\boldsymbol{R}(i,\cdot)\boldsymbol{y}^* = w^*$, where $\boldsymbol{R}(i,\cdot)$ is the i^{th} row of R matrix. This implies,

$$\sum_{j=1}^{N} O_j y_j^* - d_i O_i y_i^* = w^* = v^*,$$

hence, gives the equilibrium strategy of the adversary as equation (21).

(b) Note that $v^* = -O_k$ implies

$$\sum_{j=1}^{k} \frac{O_j - O_k}{d_j O_j} = 1,$$

in turn implies

$$1 - \sum_{j=1}^{k} \frac{1}{d_j} = -O_k \sum_{j=1}^{k} \frac{1}{d_j O_j},$$

when used in (20) gives the result (23). The primal problem is degenerate with $x_k = 0$, hence the dual problem has alternate equilibria given by (24).

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