# FORECASTING FOR INVENTORY PLANNING UNDER CORRELATED DEMAND

MELIKE BAYKAL-GÜRSOY
Department of Industrial and Systems Engineering, RUTCOR, CAIT, Rutgers University, New Brunswick, New Jersey

NESIM K. ERKIP
Department of Industrial Engineering, Bilkent University, Bilkent, Ankara, Turkey

## INTRODUCTION

When selling similar products, mainly due to product substitution by customers in case of stock outs and price/brand concerns, demand for a particular product may depend on the inventory positions of other products. Thus, the effective demand for a particular product is crosscorrelated to the demand of other products. Also, demand for each product might be autocorrelated. Demand created by advertisement or price discounts today might reduce demand in the future. From time to time, the demand might also experience other disturbances that are due to the current economic or political conditions. These dependencies make it challenging for a firm to generate accurate demand forecast for each product and to determine the "right" order quantities so as to maximize its own profit. Survey results reported at the Harvard/Wharton Merchandising Effectiveness Project [1] have found that these subjective forecasts tend to have an average forecasting error of 50% or more. As a result, some firms buy too little of some products resulting in lost sales and profit margin, and some firms buy too much of some products resulting in excess supply that must be marked down after a while, frequently to the point where the product is sold at a loss. A survey conducted by a major retailer and reported in the New York Times on June 2, 1994, concluded that 50% of customers did not purchase products when they visited the store and of these 40% stated they did so due to their inability to find a given product. Similar under- and oversupply costs have been reported for other products such as automobiles [2] and computers [3]. Demand uncertainty is highest when a new product is introduced against competition. In some industries, for example, in the pharmaceutical industry, firms prefer to be responsive to the customer demand and thus incur high inventory costs to reduce lost sale. This is where accuracy in forecasting becomes crucial.

Firms spend a great deal of time and resources trying to predict, as accurately as possible, the future demand for their products and services. Clearly, it is beneficial to know about the future, and, in most cases, the near future where accuracy is most needed. Knowing about the future enables making better decisions of ordering when inventories are reviewed—both in pull- and push-type systems. Also, production plans and resource scheduling are done more effectively in push-type systems if we have a good idea about the future.

Two issues that are of concern here are forecasting and inventory control for multi-item systems where demand for these items can be correlated, as well as temporally correlated for each item. Classical forecasting procedures are usually carried out for single items. Standard coverage of well-known text books includes the following topics: times series analysis, exponential smoothing methods (see ***Holt–Winters Exponential Smoothing***), moving average (MA) methods, regression with various possible extensions [4–8]. These references also include the study of nonstationary autoregressive integrated moving average (ARIMA) models (see ***Forecasting Nonstationary Processes***). Nevertheless, a number of procedures can be utilized for the forecasting activities under correlated demand. For example,

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regression models can be utilized to express several dependencies. However, more specialized tools, for example, multivariate time series modeling, state-space methods (see ***Forecasting: State-Space Models and Kalman Filter Estimation***), are needed for the purpose of forecasting for multiple items with correlation across items and time.

Most of the initial work done in the area of inventory theory has ignored the correlation between demand for different items as well as time periods. Instead, the demand is assumed to follow an independent stochastic process in order to ensure tractability of analytical results. However, more realistic models incorporate the past history of the market on the present demand behavior. Intuitively, it can be said that autocorrelated demand would worsen performance measures such as expected number in the warehouse and expected number of stock outs. It is important to investigate how profound the effect of demand autocorrelation is on system performance. Blinder [9] investigates the change in the optimum price and optimum order size to the demand fluctuations and shows that they will be larger when demand is highly correlated. Kahn [10], assuming a first-order autoregressive, AR(1), demand model, demonstrates that the order size is more variable than sales if demand is positively correlated. Altiok and Melamed [11] study the impact of autocorrelation on several production/inventory systems. They consider three different environments; an M/M/1 (see the section titled "Single-Station Queues: CTMC Models" in this encyclopedia) type workstation receiving autocorrelated arrival process; an M/G/1 (see the section titled "Single-Station Queues: Non-CTMC Models" in this encyclopedia) workstation with deterministic processing times, and autocorrelated operation dependent time to failures; and a single-stage manufacturing process with a raw material buffer and finished product inventory. They incorporate autocorrelation in random elements in the system, such as demand, lead time, time to failure, and repair times and investigate the effect on some system performance measures, such as average product waiting/flow times, throughput, and customer service levels. The authors conclude that finished product level and customer service level are highly affected by the autocorrelation in the demand process [11].

We think that the following questions are valid for inventory analysis:

- How can one represent the demand function(s) that consider(s) the existing correlation structure?
- How does the level of demand correlation affect order quantities and expected profits over time?
- What is the optimal inventory control policy under correlated demand?

The first question represents the forecasting aspect, and the second and third ones represent the inventory control aspect. We aim to overview the inventory approaches in order to ascertain the demand-forecasting requirements. We then review forecasting procedures that will be helpful in realizing the aforementioned inventory models.

The next section briefly discusses the previous work on single and multiple item inventory control under correlated demand. In the section titled "Evolving Forecasts Through Time", we examine the notion of evolving forecasts, including the Martingale model of forecast evolution (MMFE). We then summarize our conclusions.

## INVENTORY PLANNING UNDER CORRELATED DEMAND

The bulk of inventory theory is based on single-item problems, with known independent and identically distributed demand. For the newsvendor problem (single period), the optimal policy is given by the *critical fractile* solution for the optimal stock level $S^*$ [12] (see ***Newsvendor Models***). In the case of a normally distributed demand with mean $\mu$ and variance $\sigma^2$, the solution simplifies to

$$S^* = \mu + z\sigma,$$

where $z$ is obtained by evaluating the cumulative distribution function of the standard normal at the critical fractile. Scarf [13] has shown the optimality of $(s_t, S_t)$ policy for the dynamic inventory control problem with setup cost, $K$, under the condition that the optimal value function at each period $t$ is $K$-convex. In such a policy, it is optimal to order up to the optimal stock level $S$ whenever the inventory level drops to the reorder level, $s$, or below. If the setup cost, $K$, is zero, the optimal policy reduces to the base-stock policy that prescribes ordering up to $S$ whenever the inventory level is less than $S$. It was shown by Veinott [14] and Sobel [15] that under certain conditions the optimal base-stock policy is myopic, that is, it can be decided only by considering the current period. Myopic policies are easier to implement and thus are preferable to the more complicated time-dependent policies. Veinott [14] has extended his results on the optimality of myopic policies to the demand distributions that are independent but not identical as long as they are stochastically increasing over time.

In this section, we review inventory models with correlated demand. The subsections consider single/multi item–single location and single/multi-item–multiple locations, with correlated demand.

## Single Item–Single Location with Correlated Demand

We first review a broader group of studies referred to here as *inventory problems under nonstationary demand*. In these models, the demand distribution can vary during each period and is considered mostly in the context of single-product single-firm inventory control. Karlin and Fabens [16] introduce a Markov-modulated demand model that depends on the changing environmental factors. In this model, environmental factors are represented as the states of a Markov chain (see the section titled "Discrete Time Markov Chains" in this encyclopedia) and the demand in any given period is a random variable with a distribution function that depends on the current state of the environment. Thus, in this model, demands are independent and identically distributed as long as the environment's state remains the same. Aware of the complexity of the analysis, the authors concentrate on finding a single optimal ordering policy irrespective of the state of the environment. Song and Zipkin [17] study the continuous-time problem where the demand process is a Markov-modulated Poisson process (MMPP). In such a process, demands arrive as a Poisson process with demand rate changing according to a Markov chain. They show that the optimal ordering policy is of state-dependent $(s, S)$-type policy when there is a fixed ordering cost in addition to the linear cost (see also [18]). Treharne and Sox [19] analyze the same model when the current state of the environment is not observable. They give heuristics and present computational results. Without assuming a Markovian structure on the environment, Morton and Pentico [20] study the finite horizon, zero lead time, nonstationary problem, and derive upper and lower near-myopic bounds. They assume that demands in successive periods are independent but not necessarily identically distributed. They develop a number of heuristics and show that the best near-myopic heuristic is robust. Anupindi *et al*. [21] studying the stationary lead-time problem, obtain near-myopic bounds and give heuristics. They show that the average error increases with increase in variance of the lead-time distribution. Bollapragada and Rao [22] examine a single-product, nonstationary inventory problem with both supply and demand uncertainty, capacity limits on ordering quantities, and service-level requirements. A scenario-based stochastic program for the static, finite-horizon problem, is presented to determine replenishment orders over the horizon. A heuristic based on the first two moments of the random variables and a normal approximation is proposed.

Other than on Markov-modulated structures, there are a few studies in the area of correlated demands. In these studies, it is assumed that the demand is correlated between products as well as time periods. Although it is very difficult to obtain analytical solutions under correlated demands, it is worth studying correlation since it tells something about the new information which is observed every period. Ray [23] studies AR(1) and MA(1) demand models under stochastic lead times. By writing the moment-generating function of lead-time demand in terms of demand variance–covariance matrix, and the lead-time distribution, he obtains the first two moments of the lead-time demand. These moments are then utilized in computing the reorder level that will provide a specified service level [24]. Johnson and Thompson [25] consider a single product, periodic-review system under stationary and nonstationary demand processes with no fixed cost. There are linear holding and shortage costs and the deliveries are instantaneous. They show that under some additional assumptions on the demand process, the optimal policies are myopic. Miller [26] introduces a multiplicative AR(1)-type uncertainty on the demand process and shows the optimality of myopic base-stock policies. Lovejoy [27], assuming a more general dependent demand structure, gives conditions under which the optimal replenishment policy is myopic. In production/inventory systems, a certain amount of information becomes available as time advances from one period to the next. It is important to incorporate this available information into the planning decisions of the system. This task can be achieved by means of forecast revisions that include correlation and evolve through time. Charnes *et al*. [28] consider a periodic-review inventory replenishment model with an order-up-to policy for the case of deterministic lead times and a covariance-stationary stochastic demand process. A method is derived for setting the inventory safety stock to achieve an exact desired stock-out probability when the autocovariance function for Gaussian demand is known. Graves [29] considers a special nonstationary demand structure, where the demand follows an integrated moving average (ARIMA (0, 1, 1)) process. For this demand process, the optimal order-up-to level is characterized and implications are discussed. Urban [30] analyzes the effect of autocorrelated demand to determine reorder levels. Specifically, first-order autoregressive and moving average (ARMA) demand processes are examined. Finally, considering nonstationary, correlated and evolving stochastic demands, Levi *et al*. [31] provide an approximation algorithm to obtain computationally efficient policies with constant worst-case performance guarantees.

Another group of studies that can be placed in this category are models with inventory and/or backorder-dependent demand structures. Inventory dependent demand is a special case of the general nonstationary demand model; within this category, the backorder-dependent demand structure has attracted some attention, as it is frequently observed in practice. Argon *et al*. [32] propose a single item, periodic-review model to investigate the effects of changes in the demand process that occur after shortage realizations. They investigate a system where the demands in successive periods are deterministic, but the level is affected by the backorder realizations. Urban [33] develops a periodic-review model with products that have autocorrelated demand, as well as demands dependent on the amount of inventory displayed to the customer. Urban [34] gives a comprehensive review of inventory models with inventory-level-dependent demand.

### Single/Multi-Item Multiple Locations with Correlated Demand

In multilocation problems, demand interactions among the locations can be modeled using correlation structures. The work by Federgruen and Zipkin [35] is one of the initial studies that consider demand correlation across locations. They approach the periodic-review problem via dynamic programming to provide results in line with Clark and Scarf [36]. Erkip *et al*. [37] investigate the single-warehouse, $n$-retailer, multiechelon supply-chain model by Eppen and Schrage [38], and extend the results to the case where demand is correlated across locations as well as through time. The extension of the model to multiple items is possible, as long as the correlation structure is known. Closed-form expressions are obtained for safety stock values. Gilbert [39] presents a multistage supply-chain model that is based on the ARIMA time series models. Given an ARIMA model of consumer demand and the lead times at each stage, it is shown that the orders and inventories at each stage also follow ARIMA models, and closed-form expressions for these models are obtained. Graves and Willems [40] consider a single-product problem with a short life cycle. Given a short product life cycle, product demand is increasingly difficult to forecast and is never really stationary because the demand rate evolves over the life of the product. The problem of placing strategic safety stocks with minimum cost is studied. They extend previous results for stationary demand to the case of nonstationary demand.

Another line of research within the multiechelon inventory systems that consider demand correlations across the items is the research on postponement. Several researchers study the benefits of product and process design that calls for delaying the differentiation of products; in other words, defer the stage after which the products assume their unique identities. Note that the end products have correlated demands, and hence aggregating them in manufacturing will help reduce the expected costs. Lee and Tang [41] develop a simple model that captures the costs and benefits associated with the use of product differentiation issues and focus on products having only one point of differentiation. Garg and Tang [42] extend these results to product families which have several points of differentiation. The analysis indicates that demand correlations in addition to some other factors play an important role in determining which point of differentiation should be delayed. Aviv and Federgruen [43] characterize the benefits of delayed differentiation and quick response programs when sales forecasts are updated over time. They consider more general settings, where parameters of the demand distributions fail to be known with accuracy or where consecutive demands are correlated.

Another stream of models that are of interest is models with advance demand information (ADI). Among many recent articles, we summarize a few. DeCroix and Mookerjee [44] study a periodic-review problem in which there is an option of purchasing advance demand information at the beginning of each period. Gallego and Özer [45] model ADI through a vector of future demands and show the optimality of a state-dependent order-up-to policy. Contrary to assuming that advance demand information is always known, Tan *et al*. [46] investigate a periodic-review inventory policy with stochastic demand and stochastic advance demand information. In this model, each signal of advance demand can be treated as a potential demand. The evolution of the ADI is modeled as a Markov chain. Finally, Tan [47] considers the demand-forecasting problem in a business-to-business environment, where some customers provide information on their future orders, that are subject to change. The inaccurate information is used to specify a metric that influences forecasting.

## EVOLVING FORECASTS THROUGH TIME

Introducing updating procedures for forecasting brings the Bayesian approaches into picture. There are a number of inventory problems that incorporates the Bayesian approach in decision making. There are two types of approaches: one assumes fully observable demand and the other, partially observable demand. Scarf [48], Azoury [49], Eppen and Iyer [50], Hill [51,52], among others, have assumed fully observable demand. For example, Lariviere and Porteus [53], as an example, considered Bayesian updating with partially observed sales. There are a number of studies that consider the notion of quick response, and hence apply Bayesian updates of forecasts to planning problems. As an example, Milner and Kouvelis [54] propose a single-period inventory modeling framework, with two ordering opportunities. The second order reacts to updated demand information and potentially capitalizes on supply-chain flexibility. The authors analyze the total inventory cost of a firm for alternate demand types: the standard assumption of independent demand over the period, fashion-driven innovative products through a Bayesian model, and innovative products with evolving demand through a Martingale process (see the section titled “Martingales” in this encyclopedia). The three demand processes exhibit very different behaviors with respect to the value of the alternate forms of flexibility.

Modeling the evolution of forecasting has been studied in the literature by several researchers. Graves *et al*. [55] and Heath and Jackson [56] suggest a general descriptive model that accommodates simultaneous evolution of demand for many time periods and captures correlations between products and time periods, which is named as the *Martingale model of forecast evolution* (*MMFE*). They argue that the MMFE approach is preferred to a direct time-series approach since it can capture the potential impact of expert judgment other than past data information. Below, we give a brief description of the model.

Consider a multi-item environment. Let $D_n^j$ denote a forecast vector for item $j$ in period $n$. We assume that there are $J$ items and the forecasts are available for $M$ periods. Hence,

$$D_n^j = \left(d_{n,n}^j, d_{n,n+i}^j, \ldots, d_{n,n+M}^j, \mu^j, \mu^j, \ldots\right),$$

where $d_{n,n}^j$ is the demand realization for item $j$ in period $n$, $d_{n,n+1}^j$ is the demand forecast for period $(n+i)$ made at period $n$. It is assumed that the periods beyond the forecast horizon have a demand forecast of $\mu^j$ (stationary).

The evolution of demand forecasts from one period to the next can be modeled according to either an additive evolution model or a multiplicative one. Here we present the MMFE method through the additive model. Accordingly, the new forecast for the $(n+i)$th period demand can be written in terms of the previous forecast and an error term as

$$\begin{aligned}
d_{n+1,n+1}^j &= d_{n,n+1}^j + \epsilon_{n+1,1}^j \\
d_{n+1,n+2}^j &= d_{n,n+2}^j + \epsilon_{n+1,2}^j \\
&\vdots \\
d_{n+1,n+M+1}^j &= \mu^j + \epsilon_{n+1,M+1}^j,
\end{aligned}$$

where for $n \geq 1$, $\overline{\epsilon}_n^j = \left(\epsilon_{n,1}^j, \epsilon_{n,2}^j, \ldots, \epsilon_{n,M+1}^j\right)$ is the vector that represents the evolution of demands and forecasts from period $n-1$ to $n$. Note that the first component $\epsilon_{n,1}^j$ is the one-step ahead forecast error. The $\overline{\epsilon}_n^j$ vectors are assumed to be independent, identically distributed, multivariate normal random vectors with mean 0. The demand forecast update equation states that as we get closer to a demand period, say period $(n+1)$, by updating the forecasts for this period successively, we reduce the variability of forecast errors. These assumptions also imply that the $i$-th step-ahead demand forecast for any $j$, $d_{n,n+i}^j$, is a martingale, and it is the conditional expectation of the $(n+i)$-th period demand, given the history up to time $n$ [55,56]. The definition of $\overline{\epsilon}_n^j$ enables us to allow for correlation among future forecasts of an item, as well as cross-correlations among forecasts of different items for a given period, $n$.

The above structure holds under the assumptions stated by Heath and Jackson [56]. The methodology suggested by MMFE is useful, as it also incorporates consistent knowledge on the forecasts. Many extensions or analyses regarding MMFE are available. In the remaining part of this section we summarize some of these studies, which also have a relation to or application in the supply chain area.

Güllü [57] considers a single-item production/inventory system that incorporates forecasts into planning decisions. He uses an additive forecast evolution model, which is a special case of MMFE and is able to capture the structure of dependent and nonstationary demand. He assumes that the production capacity is limited and unsatisfied demand is backlogged. As a benchmark, he considers a standard inventory model, which assumes the same distributional properties of demand. The suggested model yields lower expected costs and inventory levels when compared to a standard inventory model. In a similar study, Güllü [58] investigates a two-echelon allocation model consisting of a depot and several retailers, again integrating forecast revisions. The depot does not hold any inventory and unsatisfied demand is backlogged. He compares this system to a standard allocation model. The standard model results in higher order-up-to levels and higher system costs. Toktay and Wein [59] study a capacitated single-item make-to-stock environment facing a stationary, correlated demand process. They utilize the MMFE approach to generate forecast revisions. The forecast updates are then incorporated into the system to obtain approximate results for the base-stock level that yields minimum expected inventory holding and backorder costs. Iida and Zipkin [60] propose a dynamic inventory model with the MMFE model. As a special case, it includes the ARMA demand model, represented by MMFE. They combine the MMFE with a linear programming model of production/distribution system in order to find an economical safety factor level and quantify the effects of forecast error on the total system cost. Lu *et al*. [61], also considering the MMFE model of demand forecast evolution in a periodic-review inventory model, develop bounds on the optimal base-stock levels as was done in Iida and Zipkin [60]. In addition, they provide bounds on the cost error of any heuristic with respect to the optimal policy, which is computationally intractable to evaluate. They give the necessary and sufficient conditions for the optimality of myopic policies.

Dong and Lee [62] revisit the serial multi-echelon inventory system of Clark and show that the structure of the optimal stocking policy of Clark and Scarf [36] holds under time-correlated demand processes using an MMFE. Then, the authors extend the approximation to an autoregressive demand model.

Aviv [63] considers a supply chain in which the underlying demand process, possibly evolves according to a vector autoregressive time series. The author proposes an adaptive inventory replenishment policy that utilizes the Kalman filter technique (see ***Forecasting: State-Space Models and Kalman Filter Estimation***).

## FORECASTING PROCEDURES FOR CORRELATED DEMAND AND APPLICATIONS

A typical forecasting process involves analyzing historical data. Historical data may exhibit trends, seasonal variations, as well as correlations. Other factors may come into the picture such as impact of other products or services in the market on the demand of the product being forecasted. Typically, the performance of the forecasting process is evaluated on the basis of the relative error, that is the percentage difference of the actual from its forecasted value, or the mean-squared forecast error.

Time series analysis techniques are utilized to carry out the forecasting process. Demand for each product is forecasted over a planning horizon in such a way that the mean-squared forecast error is minimized. Standard time series analysis techniques consider correlation structures over time, but are not effective when there are multiple items with correlated demand.

One way of avoiding forecasting demands of multiple items with cross-correlation is to aggregate the items. On the other hand, there is a need to forecast at the item level, as well. The following is the question here: Is it better to forecast the aggregate (and hence utilize a top-down approach) and then disaggregate, or to forecast the individual items directly (and hence utilize a bottom-up approach), and then the total? In any specific application, it will be hard to say which one would be better, unless ample data is available to support both approaches. Of course, the general understanding and approach are to utilize, as much as possible, the possibilities brought by the available data. Note that there are examples of both; consider Zhou *et al*. [64], where a top-down approach is utilized, and Chen *et al*. [65], which presents different (but not independent) representations for a two-item forecasting problem—an example of the bottom-up strategy.

In case we only need to consider autocorrelation, demand data can be modeled either as a stationary ARMA process or a nonstationary ARIMA process with other controllable and uncontrollable inputs [4–8]. Stationary models keep their distributional behavior in equilibrium around a constant mean. On the other hand, the nonstationary, ARIMA, models have no constant mean level over time. Nonstationary data quite often arise in industrial forecasting. Demand processes could also be modified to incorporate the effects of external factors such as economic, political, and geographic conditions. For multi-item systems with cross-correlation, multivariate time series models [66] such as a vector ARMA(p,q) could be utilized.

Aviv [63], considering a two-stage supply chain, employs state-space methods for modeling demand process as vector AR(1)(VAR(1)) time series. The minimum mean-squared error (MMSE) estimates of the demand can then be obtained recursively via the Kalman filter (see ***Forecasting: State-Space Models and Kalman Filter Estimation***) as new observations are included over time. These observations may not include full demand information. The adaptive replenishments follow a base-stock policy that depends on the aggregate demand estimate during lead time.

In order to forecast intermittent demand, Croston [67] presents two separate exponential smoothing forecasts, one on the demand size and the other on the demand interarrival times. Several studies demonstrate that Croston's method performs better than some competing approaches [68,69]. Shenstone and Hyndman [70] attempt to identify stochastic models that underlie Croston's method in order to obtain confidence intervals for the forecasts. One such model assumes a nonstationary ARIMA(0,1,1) time series model for both the demand size and the demand interarrival times.

There are a few studies that combine forecasting performance with inventory performance. The difficulty lies in creating benchmarks, and we believe that more work in this area is needed.

One example is by Sani and Kingsman [69] who compare various inventory replenishment heuristics of $(s, S)$ type in addition to the comparison of forecasting methods for a low demand item.

Chen *et al*. [65] report another study that combines forecasting and inventory control. The authors use a bivariate VAR(1) time-series model to investigate the effects of aggregating two interrelated demands. The paper further explores the properties of the aggregated time series and provides guidelines for practitioners to determine proper aggregation and forecasting approaches. A method is proposed to estimate the parameters of the demand model.

Other examples are from the production planning environments where forecast procedures are integrated with the planning procedures. Heath and Jackson [56] combine the MMFE with a linear programming model of production/distribution system in order to find an economic safety stock level and quantify the effects of forecast error on the total system cost. A similar study is carried out by Kayhan *et al*. [71], where the application of MMFE to an industrial case is considered. Procedures to set safety stocks for an LP (linear programming)-based production planning model are proposed utilizing forecast error correlations across items and time modeled by MMFE.

## CONCLUSIONS

In this article, we consider single-/multi-item supply chains under stationary or nonstationary correlated demand. We review inventory approaches in order to establish the demand-forecasting requirements. We also discuss the notion of evolving forecasts, including MMFE. We summarize various approaches for forecasting the demand of such inventory systems.

Forecasting is a very important tool for supply chains. Customer behavior and retail performance are factors affecting the demand for products. More comprehensive approaches for forecasting are needed that will consider all aspects of demand. On the other hand, we observe that there are still challenges ahead in achieving well-performing plans in a dynamically changing, uncertain environment, for more classical forecasting.

## REFERENCES

1. Fisher M, Raman A. Survey results reported at the Harvard/Wharton Merchandising Effectiveness Project; 1998.
2. White JB. Hughes charge give GM net loss of 357 million. Wall Street Journal. 1992 August 7.
3. Stewart TA. Brace for Japan's hot new strategy. Fortune. 1992 Sep 21.
4. Box GEP, Jenkins GM, C G. Reinsel. Time series analysis: forecasting and control. 3rd ed. Englewood Cliffs (NJ): Prentice Hall; 1994.
5. Brockwell PJ, Davis RA. Time series: theory and methods. 2nd ed. New Jersey: Springer; 1991.

6. Brockwell PJ, Davis RA. Introduction to time series and forecasting. 2nd ed. New Jersey: Springer; 2002.
7. Montgomery DC, Jennings CL, Kulahci M. Introduction to time series analysis and forecasting. New York: John Wiley & Sons, Inc.; 2008.
8. Makridakis SG, Wheelwright SC, Hyndman RJ. Forecasting: methods and applications. 3rd ed. New York: John Wiley & Sons, Inc.; 1998.
9. Blinder A. Inventories and sticky prices: more on the microfoundations of macroeconomics. Am Econ Rev 1982;72(3):334–348.
10. Kahn JA. Inventories and the volatility of production. Am Econ Rev 1987;77(4):667–679.
11. Altiok T, Melamed B. The case for modeling correlation in manufacturing systems. IIE Trans 2001;33:779–791.
12. Arrow K, Harris T, Marschak J. Optimal inventory policy. Econometrica 1951;19:250–272.
13. Scarf H. The optimality of (s, S) policies in the dynamic inventory problem. In: Arrow J, Karlin S, Suppes P, editors. Mathematical methods in the social sciences. Stanford (CA): Stanford University Press; 1960: pp. 196–202.
14. Veinott AF. Optimality policy for multi-product, dynamic, nonstationary inventory problem. Manage Sci 1965;12:206–222.
15. Sobel MJ. Myopic solutions of Markov decision processes and stochastic games. Oper Res 1981;29:995–1009.
16. Karlin S, Fabens A. The (s, S) inventory model under Markovian demand process. In: Arrow J, Karlin S, Suppes P, editors. Mathematical methods in the social sciences. Stanford (CA): Stanford University Press; 1960.
17. Song J-S, Zipkin P. Inventory control in a fluctuating demand environment. Oper Res 1993;41:351–370.
18. Sethi SP, Cheng F. Optimality of (s, S) policies in inventory models with Markovian demand. Manage Sci 1997;45:931–939.
19. Treharne JT, Sox CR. Adaptive inventory control for nonstationary demand and partial information. Manage Sci 2002;48:607–624.
20. Morton TE, Pentico D. The finite horizon nonstationary stochastic inventory problem: near-myopic bounds, heuristics, testing. Manage Sci 1995;41:334–343.
21. Anupindi R, Morton TE, Pentico D. The nonstationary stochastic lead-time inventory problem: Near-myopic bounds, heuristics, and testing. Manage Sci 1996;42:124–129.
22. Bollapragada R, Rao US. Replenishment planning in discrete-time, capacitated, non-stationary, stochastic inventory systems. IIE Trans 2006;38(7):583–595.
23. Ray WD. The significance of correlated demands and variable lead times for stock control policies. J Oper Res Soc 1980;31(2): 187–190.
24. Ray WD. Computation of reorder levels when the demands are correlated and the lead time random. J Oper Res Soc 1981;32(1): 27–34.
25. Johnson GD, Thompson HE. Optimality of myopic inventory policies for certain dependent demand processes. Manage Sci 1975;21:1303–1307.
26. Miller BL. Scarf's state reduction method, flexibility, and a dependent demand inventory model. Oper Res 1986;34(1):83–90.
27. Lovejoy WS. Myopic policies for some inventory models with uncertain demand distributions. Manage Sci 1990;36(6):724–738.
28. Charnes JM, Marmorstein H, Zinn W. Safety stock determination with serially correlated demand in a periodic-review inventory system. J Oper Res Soc 1995;46(8):1006–1013.
29. Graves SC. A single-item inventory model for a non-stationary demand process. Manuf Serv Oper Manage 1999;1(1):50–61.
30. Urban TL. Reorder level determination with serially-correlated demand. J Oper Res Soc 2000;51(6):762–768.
31. Levi R, Pál M, Roundy RO, *et al*. Approximation algorithms for stochastic inventory control models. Math Oper Res 2007;32(2): 284–302.
32. Argon NT, Güllü R, Erkip N. Analysis of an inventory system under backorder correlated deterministic demand and geometric supply process. Int J Prod Econ 2001;71(1-3): 247–254.
33. Urban TL. A periodic-review model with serially-correlated, inventory-level-dependent demand. Int J Prod Econ 2005;95(3):287–295.
34. Urban TL. Inventory models with inventory-level-dependent demand: a comprehensive review and unifying theory. Eur J Oper Res 2005;162(3):792–804.
35. Federgruen A, Zipkin P. Computational issues in an infinite-horizon, multi-echelon inventory model. Oper Res 1984;32(4):818–836.

36. Clark A, Scarf H. Optimal policies for a multi-echelon inventory problem. Manage Sci 1960;6:475–490.
37. Erkip NK, Hausman WH, Nahmias S. Optimal centralized ordering policies in multi-echelon inventory systems with correlated demands. Manage Sci 1990;36(3):381–392.
38. Eppen G, Schrage L. Centralized ordering policies in a multi-warehouse system with leadtimes and random demand. In: Schwartz L, editor. Multi-level Production/Inventory control systems: theory and practice. Amsterdam: North-Holland; 1981. pp. 51–69.
39. Gilbert K. An ARIMA supply chain model. Manage Sci 2005;51(2):305–310.
40. Graves SC, Willems SP. Strategic inventory placement in supply chains: Non-stationary demand. Manuf Serv Oper Manage 2008; 10(2):278–287.
41. Lee HL, Tang CS. Modeling the costs and benefits of delayed product differentiation. Manage Sci 1997;53(1):40–53.
42. Garg A, Tang CS. On postponement strategies for product families with multiple points of differentiation. IIE Trans 1997;29(8):641–650.
43. Aviv Y, Federgruen A. Design for postponement: a comprehensive characterization of its benefits under unknown demand distributions. Oper Res 2001;49(4):578–598.
44. DeCroix GA, Mookerjee VS. Purchasing demand information in a stochastic-demand inventory. Eur J Oper Res 1997;102:36–57.
45. Gallego G, Özer O. Integrating replenishment decisions with advance demand information. Manage Sci 2001;47:1344–1360.
46. Tan TR, Güllü R, Erkip N. Modelling imperfect advance demand information and analysis of optimal inventory policies. Eur J Oper Res 2007;177:897–923.
47. Tan TR. Using imperfect advance demand information in forecasting. IMA J Manage Math 2008;19:163–173.
48. Scarf H. Some remarks on Bayes solutions to the inventory problem. Nav Res Log Q 1960; 7:591–596.
49. Azoury K. Bayes solution to dynamic inventory models under unknown demand distribution. Manage Sci 1985;31:1150–1160.
50. Eppen GD, Iyer AV. Improved fashion buying with bayesian updates. Oper Res 1997;45: 805–819.
51. Hill RM. Applying bayesian methodology with a uniform prior to the single period inventory model. Eur J Oper Res 1997;98:555–562.
52. Hill RM. Bayesian decision-making in inventory modeling. IMA J Math Appl Bus Ind 1999;10:147–163.
53. Lariviere MA, Porteus LE. Stalking information: Bayesian inventory management with unobserved lost sales. Manage Sci 1999;45: 346–363.
54. Milner JM, Kouvelis P. Order quantity and timing flexibility in supply chains: The role of demand characteristics. Manage Sci 2005;51(6):970–985.
55. Graves SC, Meal HC, Dasu S, *et al*. Two-stage production planning in a dynamic environment. In: Schneeweiss C, Axsater S, Silver E, editors. Volume 266, Multi-stage production planning and inventory control, Lecture notes in economics and mathematical systems. Berlin: Springer; 1986. pp. 9–43.
56. Heath DC, Jackson PL. Modeling the evolution of demand forecasts with application to the to safety stock analysis in production/distribution systems. IEE Trans 1994;26(3):17–30.
57. Güllü R. On the value of information in dynamic production/inventory problems under forecast evolution. Nav Res Log 1996; 43:289–303.
58. Güllü R. A two-echelon allocation model and the value of information under correlated forecasts and demands. Eur J Oper Res 1997; 99:386–400.
59. Toktay LB, Wein LM. Analysis of forecasting-production-inventory system with stationary demand. Manage Sci 2001;47(9):1268–1281.
60. Iida T, Zipkin P. Approximate solutions of a dynamic forecast-inventory model. Manuf Serv Oper Manage 2006;8(4):407–425.
61. Lu X, Song J-S, Regan A. Inventory planning with forecast updates: Approximate solution and cost error bounds. Oper Res 2006;54(6):1079–1097.
62. Dong LX, Lee HL. Optimal policies and approximations for a serial multiechelon inventory system with time-correlated demand. Oper Res 2003;51(6):969–980.
63. Aviv Y. A time-series framework for supply-chain inventory management. Oper Res 2003;51(2):210–227.
64. Zhou S, Jackson PL, Roundy RO, *et al*. The evolution of family level sales forecasts into product level forecasts: Modeling and estimation. IIE Trans 2007;39(9):831–843.
65. Chen A, Hsu CH, Blue J. Demand planning approaches to aggregating and forecasting interrelated demands for safety stock and

backup capacity planning. Int J Prod Res 2007;45(10):2269–2294.

66. Reinsel GC. Elements of multivariate time series analysis. New York: Springer; 1993.

67. Croston JD. Forecasting and stock control for intermittent demands. Oper Res Q 1972; 23(3):289–303.

68. Willemain TR, Smart CN, Shocker JH, *et al*. Forecasting intermittent demand in manufacturing:A comparative evaluation of Croston's method. Int J Forecast 1994;10:529–538.

69. Sani B, Kingsman BG. Selecting the best periodic inventory control and demand forecasting methods for low demand items. J Oper Res Soc 1997;48(7):700–713.

70. Shenstone L, Hyndman RJ. Stochastic models underlying Croston's method for intermittent demand forecasting. J Forecast 2005; 24:389–402.

71. Kayhan M, Güllü R, Erkip N. Integrating forecast evolution with production/inventory planning: an implementation framework for a fast moving consumer goods manufacturer. Working Paper, Ankara: Middle East Technical University; 2005.